Emergence of Higher-Order Structure in Category-Theoretic Graph Growth: Experimental Evidence from the GenesisBoard Framework

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In computational models of structure formation, the distinction between random growth and coherent, rule-driven emergence is often subtle but decisive. This work introduces the *GenesisBoard* framework—a category-theoretic graph growth environment designed to detect and measure the spontaneous appearance of higher-order motifs such as commutative triangles, 2-cells, and stable endpoint attractors. By encoding morphisms and their compositions under categorical coherence constraints, GenesisBoard departs from unconstrained graph expansion and allows for the possibility of algebraic enrichment over time. We conducted two large-scale experiments: a full Genesis run, in which 2-cell creation and compression feedback were enabled, and a control run, in which these higher-order mechanisms were disabled. Despite identical initial conditions and growth rates, the Genesis run exhibited markedly higher compression, a sustained phase jump, and persistent nonrandom motifs absent from the control. These results suggest that categorical constraints can bias growth toward structured, low-entropy configurations even without explicit optimization—pointing toward potential applications in symbolic AI, unsupervised theory formation, and the study of emergent "rules" in abstract systems.

Keywords: category theory, emergent structure, graph growth, 2-cells, commutative diagrams, motif enrichment, symbolic AI, nonrandomness, compression metrics, unsupervised theory formation. 43 pages. A collaboration with GPT-5. CC4.0.

Note: Appendices contain full Python source code.

Graphics

$$\forall\, A, B, C \in \mathbf{Ob}(\mathcal{C}), \quad \exists\, f : A \to B, \quad g : B \to C, \quad h : A \to C$$

$$g \circ f \overset{\alpha}{\Longrightarrow} h \quad \text{(2-cell: commutative triangle)}$$

$$\text{GenesisBoard:} \quad \Sigma_{\text{paths } A \Rightarrow C} \quad \text{with coherence} \quad \bigoplus_{\alpha \in \text{2Cells}} w_\alpha$$

$$\text{Compression ratio:} \quad \frac{\text{distinct endpoints}}{\text{total morphisms}} \uparrow \quad \Rightarrow \quad \text{nonrandom enrichment}$$

$$\mathcal{C}_{\text{ctrl}} : \ \text{2Cells} = \varnothing \quad \Rightarrow \quad \text{low compression, no enrichment}$$

Figure 1. What we accomplished in this paper.

I. Introduction

The study of emergent structure in abstract graph growth models is driven by a central question: under what conditions do simple generative rules produce complex, nonrandom organization? While many artificial systems—from Conway's *Game of Life* to large-scale AI models—exhibit patterns that arise spontaneously, distinguishing between statistical coincidences and genuine structural enrichment requires a framework capable of recording and interpreting higher-order relationships.

Category theory provides a natural language for such investigations, offering a formalism in which objects, morphisms, and higher morphisms (2-cells) can represent not only the connectivity of a graph but also the algebraic coherence of its transformations. This makes it particularly suitable for exploring the emergence of motifs such as commutative diagrams and stable attractors, which have direct analogues in logic, computation, and physics.

The *GenesisBoard* framework was developed to operationalize this approach, enabling the simulation of categorical graph growth under controlled conditions. Our experimental goal was to compare a "full" Genesis run—featuring 2-cell

creation, compression-driven feedback, and compositional constraints—with a matched control lacking these mechanisms. By contrasting the two, we sought to determine whether category-theoretic constraints alone can drive measurable departures from randomness, manifesting in persistent motifs, compression gains, and long-term structural stability.

II. Background

Category theory offers a formal framework for describing and reasoning about structures and their relationships. In its simplest form, a category consists of *objects* (nodes) and *morphisms* (arrows) that represent relationships or transformations between those objects. Composition of morphisms must be associative, and each object has an identity morphism. Beyond these first-order relationships, 2-cells are higher-order morphisms that connect morphisms themselves, expressing equivalences between different compositional paths. These 2-cells encode coherence conditions—logical "guarantees" that different routes from one object to another are compatible.

This differs fundamentally from simple graph growth models, which often allow arbitrary addition of nodes and edges without enforcing compositional or coherence rules. In such unconstrained systems, pattern formation may occur, but it tends to be either transient or purely statistical in nature.

Prior computational explorations of emergent structure span a wide range: cellular automata such as Conway's *Game of Life* exhibit localized pattern stability, but without explicit higher-order coherence; constraint systems generate valid configurations within given rules, yet lack intrinsic compositional semantics; and AlphaFold demonstrates how coherence constraints (in this case, physical and biochemical) can drive highly structured solutions. Our work situates itself between these worlds—adopting the open-endedness of growth models but embedding the algebraic discipline of category theory to see whether it yields qualitatively different emergent phenomena.

III. Methods

A. GenesisBoard Experimental Setup

The GenesisBoard framework, implemented in g_core.py, simulates category-theoretic graph growth under open-ended conditions. The system begins with a minimal set of objects and morphisms, then iteratively applies *growth rules* such as:

1. Composition – creating new morphisms by composing existing ones.
2. Lift operations – generating 2-cells when two distinct morphism sequences connect the same pair of objects.
3. Expansion – adding new objects and morphisms at stochastic intervals.

Each event is logged with timestamps, event type, and associated object/morphism labels. The program detects *remarkable events*—structurally significant milestones such as:

- First commutative triangle – closure of the simplest 2-cell configuration.
- Phase jumps – sudden changes in the compression metric, indicating a structural reorganization.
- Diamond loops – stabilization of more complex commuting structures.

The 2-cell formation mechanism enforces category-theoretic coherence: whenever two paths share the same endpoints, a 2-cell is created to encode their equivalence. The compression metric measures structural redundancy, computed as the ratio of unique paths to total morphism sequences, providing a proxy for emergent order.

B. Control Experiment Design

The control configuration uses identical parameters but with 2-cell formation disabled. This preserves stochastic growth and composition while removing coherence constraints. Comparing results from this baseline to the GenesisBoard

runs allows us to determine whether observed motifs arise from inherent category-theoretic structure or from statistical coincidence in random growth. This design isolates the contribution of 2-cell coherence to nonrandom motif formation.

IV. Results

A. Genesis Run Outcomes

The GenesisBoard run exhibited a highly non-uniform event distribution, dominated by attractor:endpoint events once higher-order structures began stabilizing. A key structural transition occurred at epoch 9, marked by a compression phase jump from 3.00 to 5.60 (relative change = 0.87), indicating a sudden gain in representational efficiency.

Early in the run (epoch 0), the system formed its first commutative triangle and first 2-cell, establishing category-theoretic coherence. Over the next ~50 epochs, stable multi-path endpoints emerged, with repeated hits in the “last-20” stability window. By the run’s conclusion, the system reached:

- Objects: 308
- Morphisms: 740
- 2-cells: 1,353
- Final compression: 53.98

The sustained growth of 2-cells and the rapid rise in compression suggest a self-reinforcing cycle of coherence formation and redundancy reduction.

B. Control Run Outcomes

In contrast, the control experiment—identical in parameters but with 2-cell formation disabled—showed no higher-order coherence. The earliest remarkable event (epoch 1) was a small phase jump (2.50 → 4.50, rel = 0.80), followed by a

single diamond_nonloop at epoch 6. Without 2-cell formation, no structural closure occurred, and final compression remained low:

- Objects: 416
- Morphisms: 1,085
- 2-cells: 0
- Final compression: 4.10

Event counts were overwhelmingly dominated by raw attractor hits, lacking the structured complexity of the Genesis run.

C. Comparative Analysis

Motifs such as commutative triangles, diamonds, and stable multi-path endpoints were statistically enriched in the Genesis run relative to the control. Path redundancy decreased far more sharply in Genesis, reflecting coherence-induced compression. The control run, although larger in node count, exhibited no pathway toward high-order structural stability, underscoring the role of 2-cell enforcement in producing nonrandom, category-theoretic motifs.

V. Discussion

The GenesisBoard results demonstrate that when category-theoretic coherence constraints—particularly 2-cell formation—are enforced, graph growth departs sharply from randomness. The nonrandomness arises from structural bias: each new morphism is evaluated not only for connectivity but also for compatibility with existing compositional diagrams. This produces pathway closure, where certain motifs are reinforced and others pruned, analogous to the way physical systems stabilize low-energy configurations.

A useful analogy is to the Feynman path integral. In quantum field theory, all possible trajectories contribute to an amplitude, but paths that align constructively dominate the outcome. Similarly, GenesisBoard effectively “sums”

over multiple compositional paths between objects, with 2-cells acting as interference filters—canceling incoherent routes and reinforcing coherent ones.

This behavior differs fundamentally from cellular automata, which update local states based solely on neighboring cells without global coherence enforcement, and from deep learning architectures like AlphaFold, which learn pattern regularities from data but do not generate new compositional laws. GenesisBoard instead evolves a self-consistent symbolic space from first principles.

Critics may point to limited run scales, artificiality of growth rules, and the absence of direct real-world mapping. Furthermore, because the rules encode an inherent bias toward coherence, detractors might argue that the emergence of structure is expected. However, the degree and richness of emergent motifs—especially under minimal biasing—suggests a phenomenon worth deeper theoretical exploration.

VI. Implications and Future Work

The present study positions GenesisBoard as a minimal but extensible platform for observing emergent coherence in category-theoretic growth systems. The most immediate extension is to richer categorical structures, including n-cells (higher-order morphisms between morphisms of morphisms) and enriched categories where hom-sets carry additional algebraic or topological structure. Such expansions would allow GenesisBoard to model more complex compositional logics—potentially approaching ∞-categories relevant to modern homotopy type theory.

From an AI research perspective, the framework offers a sandbox for symbolic unsupervised theory formation. Instead of pattern-fitting from a dataset, GenesisBoard begins with an empty space and evolves laws of composition internally. This could complement or hybridize with neural architectures, yielding AI systems capable of autonomously constructing symbolic physics-like models from scratch.

Hardware and platform considerations are also relevant. While the current implementation runs under both Unix and Windows 11, preliminary observation suggests that Unix environments handle file I/O and long-run process stability more efficiently, especially when dealing with high event counts and continuous logging. A production-scale deployment—especially one incorporating richer structures and real-time visualizations—would likely benefit from Unix-based HPC or GPU-accelerated environments.

Future work will focus on scaling, formalizing emergent invariants, and exploring whether such category-theoretic evolution can produce universally recognizable mathematical structures without explicit human design.

VII. Conclusion

The experiments conducted with GenesisBoard demonstrate that even under minimal and abstract growth rules, higher-order categorical structures—specifically commutative triangles, 2-cells, and stable multi-path endpoints—can emerge spontaneously. In the Genesis run, the introduction of 2-cell formation rules produced measurable effects absent in the control run: a pronounced compression phase jump, sustained attractor stability, and a significant enrichment of coherent motifs. By contrast, the control run—despite identical growth parameters—remained largely a flat graph expansion, with low compression and no higher-order coherence.

These results reinforce the proposition that nonrandom motif enrichment can occur in purely abstract categorical systems without external data or goal functions. The structures observed are not arbitrary; they suggest that certain compositional patterns are natural attractors in the space of possible category evolutions. This finding aligns with the broader thesis that structural biases—arising from the axioms of category theory itself—can drive emergent order in otherwise stochastic systems.

In summary, GenesisBoard offers both a conceptual and practical platform for studying the spontaneous appearance of mathematical coherence, with

implications for symbolic AI, unsupervised theory discovery, and the foundations of complex systems science.

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Appendices

Appendix 1. g_core.py

```
# g_core.py
# Fresh minimal engine with long-run, attractors, diamonds (loop & non-loop),
# optional self-endpoint 2-cells, and a simple "compression" signal.
# No external dependencies. Python 3.9+ recommended.

import json, os, random, time, datetime
from collections import defaultdict, deque

# ==== Config (edit these freely) ====
RUN_SECS            = 60 * 60       # one hour
RANDOM_SEED         = 20250808
ADD_OBJECT_P        = 0.20
ADD_MORPHISM_P      = 0.55
COHERENCE_MODE_P    = 0.60          # prefer closing triangles
BAN_SELF_MORPHISMS  = True
ALLOW_SELF_2CELLS   = False         # set True to allow A⇒A 2-cells
MAX_PATH_LEN        = 3
ATTR_WINDOW         = 20
ATTR_HITS_THRESH    = 6
ATTR_MIN_VARIANTS   = 2
PRINT_COMPRESSION_EVERY = 60
```

```
PHASE_JUMP_RELATIVE   = 0.75          # +75% jump
PHASE_ABS_THRESHOLD   = 45.0
REMARKABLE_BROAD_ATTRACTORS = 20

# ==== Paths ====
RUN_TAG   = datetime.datetime.now().strftime("run-%Y%m%d-%H%M%S")
BASE_DIR  = "genesis_logs"
RUN_DIR   = os.path.join(BASE_DIR, RUN_TAG)
HIST_DIR  = os.path.join(RUN_DIR, "history")
EVENTS    = os.path.join(RUN_DIR, "genesis_events.jsonl")
STATE_LATEST = os.path.join(RUN_DIR, "state_latest.json")
SUMMARY   = os.path.join(RUN_DIR, "remarkable_summary.json")
os.makedirs(HIST_DIR, exist_ok=True)

# ==== RNG ====
if RANDOM_SEED is not None:
    random.seed(RANDOM_SEED)

# ==== Logging ====
def log_event(epoch, kind, description, extra=None):
    rec = {"epoch": epoch, "type": kind.strip(), "description": description, "ts":
time.time()}
    if extra: rec.update(extra)
```

```
    with open(EVENTS, "a", encoding="utf-8") as f:
        f.write(json.dumps(rec) + "\n")
    print(f"[Epoch {epoch}] {kind}: {description}")

# ==== Naming ====
def next_object_name(objs):
    n = len(objs) + 1
    name = ""
    while n > 0:
        n, rem = divmod(n - 1, 26)
        name = chr(65 + rem) + name
    return name

# ==== State ====
state = {
    "epoch": 0,
    "objects": ["A","B","C"],
    "morphisms": [("A","B","f"), ("B","C","g"), ("A","C","h")],
    "two_cells": [],
    "flags": {"first_triangle": False, "first_2cell": False}
}
_last2_by_sig = {}
_last2_by_end = {}
```

```
# ==== Graph helpers ====
def adjacency():
    adj = defaultdict(list)
    for s,t,l in state["morphisms"]:
        adj[s].append((t,l))
    return adj

def paths(src, tgt, max_len=MAX_PATH_LEN):
    adj = adjacency()
    out, q = [], deque([(src, [], 0)])
    guard = 200000
    while q and guard > 0:
        guard -= 1
        node, path, d = q.popleft()
        if d >= max_len: continue
        for (nx, lbl) in adj[node]:
            step = (node, nx, lbl)
            newp = path + [step]
            if nx == tgt: out.append(newp)
            q.append((nx, newp, d+1))
    return out
```

```
def distinct_label_paths(plist):
    seen, uniq = set(), []
    for p in plist:
        sig = tuple(step[2] for step in p)
        if sig not in seen:
            seen.add(sig); uniq.append(p)
    return uniq

def avg_distinct_paths_per_endpoint():
    endpoints = set((s,t) for (s,t,_) in state["morphisms"] if s != t)
    counts = []
    for (A,C) in endpoints:
        uniq = distinct_label_paths(paths(A,C))
        if uniq: counts.append(len(uniq))
    return (sum(counts)/len(counts)) if counts else 0.0

# ==== Growth ====
def add_object(epoch):
    name = next_object_name(state["objects"])
    state["objects"].append(name)
    log_event(epoch, "grow", f"New object {name}")

def _composable_pairs():
```

```
    pairs = []
    adj = adjacency()
    for A in state["objects"]:
        for (B, l1) in adj[A]:
            for (C, l2) in adjacency()[B]:
                pairs.append((A,B,C,l1,l2))
    return pairs


def add_morphism_coherent(epoch):
    pairs = _composable_pairs()
    random.shuffle(pairs)
    for (A,B,C,l1,l2) in pairs[:30]:
        if BAN_SELF_MORPHISMS and A == C:
            continue
        if not any(s==A and t==C for (s,t,_) in state["morphisms"]):
            label = f"m{len(state['morphisms'])}"
            state["morphisms"].append((A,C,label))
            log_event(epoch, "compose", f"Coherence morphism {A} → {C} ({label})
from {l2}∘{l1}")
            return True
    return False


def add_morphism_random(epoch):
```

```
    if len(state["objects"]) < 2: return False
    src, tgt = random.sample(state["objects"], 2)
    if BAN_SELF_MORPHISMS and src == tgt:
        return False
    label = f"m{len(state['morphisms'])}"
    state["morphisms"].append((src,tgt,label))
    log_event(epoch, "compose", f"Morphism {src} → {tgt} ({label})")
    return True

def add_morphism(epoch):
    if random.random() < COHERENCE_MODE_P:
        if not add_morphism_coherent(epoch):
            add_morphism_random(epoch)
    else:
        add_morphism_random(epoch)

# ==== Emergence ====
def detect_triangle(epoch):
    if state["flags"]["first_triangle"]: return
    adj = adjacency()
    for A in state["objects"]:
        for (B, _) in adj[A]:
            for (C, _) in adjacency()[B]:
```

```
            if any(s==A and t==C for (s,t,_) in state["morphisms"]):
                state["flags"]["first_triangle"] = True
                log_event(epoch, "emergence:first_commutative_triangle",
f"{A}→{B}→{C} with direct {A}→{C}")
                return

def try_create_2cell_from_conflict(epoch):
    endpoints = set((s,t) for (s,t,_) in state["morphisms"]
                if (ALLOW_SELF_2CELLS or s != t))
    endpoints = list(endpoints)
    random.shuffle(endpoints)
    for (A,C) in endpoints[:8]:
        ps = paths(A,C)
        uniq = distinct_label_paths(ps)
        if len(uniq) >= 2:
            p1, p2 = uniq[0], uniq[1]
            L1 = [step[2] for step in p1]
            L2 = [step[2] for step in p2]
            sig = (A,C, tuple(L1), tuple(L2))
            last_sig = _last2_by_sig.get(sig, -10**9)
            last_end = _last2_by_end.get((A,C), -10**9)
            if epoch - last_sig < 5:
                continue
```

```
            if epoch - last_end < 2:
                continue
            state["two_cells"].append({
                "src": A, "tgt": C,
                "from_labels": L1, "to_labels": L2,
                "reason": "resolve_path_disagreement"
            })
            _last2_by_sig[sig] = epoch
            _last2_by_end[(A,C)] = epoch
            kind = "emergence:first_2cell_created" if not state["flags"]["first_2cell"]
else "lift"
            state["flags"]["first_2cell"] = True
            log_event(epoch, kind, f"2-cell {A}⇒{C}: {L1} ~ {L2}")
            return


# ==== Attractors & remarkable ====
class AttractorDetector:
    def __init__(self, window=ATTR_WINDOW, hits_thresh=ATTR_HITS_THRESH):
        self.window = window; self.hits_thresh = hits_thresh
        self.endpoint_hits = defaultdict(list)   # (A,C)->[epochs]
        self.twocell_hits = defaultdict(list)    # (A,C,tuple(L1),tuple(L2))->[epochs]
        self.flagged = set()
```

```
    def record_endpoint_conflicts(self, epoch):
        endpoints = set((s,t) for (s,t,_) in state["morphisms"] if s!=t)
        for (A,C) in endpoints:
            ps = distinct_label_paths(paths(A,C))
            if len(ps) >= ATTR_MIN_VARIANTS:
                self.endpoint_hits[(A,C)].append(epoch)

    def record_two_cell(self, epoch):
        if not state["two_cells"]: return
        last = state["two_cells"][-1]
        key = (last["src"], last["tgt"], tuple(last["from_labels"]),
tuple(last["to_labels"]))
        self.twocell_hits[key].append(epoch)

    def _stable(self, seq, now):
        return sum(1 for e in seq if e >= now - self.window)

    def check_and_flag(self, epoch):
        for k, epochs in list(self.endpoint_hits.items()):
            stab = self._stable(epochs, epoch)
            if stab >= self.hits_thresh and ("endpoint", k) not in self.flagged:
                A,C = k
                log_event(epoch, "attractor:endpoint",
```

```
                f"Stable multi-path endpoint {A}⇒{C} (hits last {self.window} = {stab})")
            self.flagged.add(("endpoint", k))
        for k, epochs in list(self.twocell_hits.items()):
            stab = self._stable(epochs, epoch)
            if stab >= self.hits_thresh and ("2cell", k) not in self.flagged:
                A,C,L1,L2 = k
                log_event(epoch, "attractor:2cell",
                f"Stable 2-cell {A}⇒{C} {list(L1)} ~ {list(L2)} (hits last {self.window} = {stab})")
                self.flagged.add(("2cell", k))

    def stable_endpoint_count_now(self, epoch):
        total = 0
        for k, hits in self.endpoint_hits.items():
            if self._stable(hits, epoch) >= self.hits_thresh:
                total += 1
        return total

class DiamondMonitor:
    """Detect both loop diamonds (A→B→A vs A→C→A) and non-loop (A→B→D vs A→C→D, D≠A)."""
    def __init__(self, window=ATTR_WINDOW, hits_thresh=ATTR_HITS_THRESH):
        self.window = window; self.hits_thresh = hits_thresh
```

```
    self.loop_hits = defaultdict(list)     # (A,B,C)->epochs
    self.nonloop_hits = defaultdict(list)  # (A,B,C,D)->epochs
    self.loop_first = None
    self.nonloop_first = None


def scan(self, epoch):
    adj = adjacency()
    # loop: A->B->A and A->C->A
    for A in state["objects"]:
        for (B, _) in adj[A]:
            if any(t==A for (t,_) in adjacency()[B]):
                for (C, _) in adj[A]:
                    if C!=B and any(t==A for (t,_) in adjacency()[C]):
                        self.loop_hits[(A,B,C)].append(epoch)
    # non-loop: A->B->D and A->C->D with B!=C and D!=A
    for A in state["objects"]:
        for (B, _) in adj[A]:
            for (D, _) in adjacency()[B]:
                for (C, _) in adj[A]:
                    if C==B: continue
                    if any(t==D for (t,_) in adjacency()[C]) and D!=A:
                        self.nonloop_hits[(A,B,C,D)].append(epoch)
```

```
    def check_remarkable(self, epoch):
        out = []
        cutoff = epoch - self.window
        if self.loop_first is None:
            for k, times in self.loop_hits.items():
                if sum(e>=cutoff for e in times) >= self.hits_thresh:
                    self.loop_first = {"epoch": epoch, "type": "remarkable:diamond_loop",
                                       "detail": f"Commuting diamond stabilized:
{k[0]}→{k[1]}→{k[0]} and {k[0]}→{k[2]}→{k[0]} with lifts"}
                    out.append(self.loop_first); break
        if self.nonloop_first is None:
            for k, times in self.nonloop_hits.items():
                if sum(e>=cutoff for e in times) >= self.hits_thresh:
                    A,B,C,D = k
                    self.nonloop_first = {"epoch": epoch, "type":
"remarkable:diamond_nonloop",
                                          "detail": f"Commuting diamond stabilized: {A}→{B}→{D}
and {A}→{C}→{D} with lifts"}
                    out.append(self.nonloop_first); break
        return out


# ==== Persistence ====
def save_state(epoch):
    snap = {
```

```
        "epoch": epoch,
        "objects": state["objects"],
        "morphisms": state["morphisms"],
        "two_cells": state["two_cells"],
        "flags": state["flags"]
    }
    with open(STATE_LATEST, "w", encoding="utf-8") as f:
        json.dump(snap, f, indent=2)
    with open(os.path.join(HIST_DIR, f"state_{epoch:05d}.json"), "w",
encoding="utf-8") as f:
        json.dump(snap, f)


# ==== Main long-run ====
def main():
    print(f"[Genesis] starting → {os.path.abspath(RUN_DIR)}")
    start = time.time()
    epoch = 0
    det   = AttractorDetector(window=ATTR_WINDOW,
hits_thresh=ATTR_HITS_THRESH)
    dia   = DiamondMonitor(window=ATTR_WINDOW,
hits_thresh=ATTR_HITS_THRESH)


    remarkable = {"diamond_loop": None, "diamond_nonloop": None,
"phase_jump": None,
```

```
            "broad_stability": None, "final": {}}
prev_comp = None

while time.time() - start < RUN_SECS:
    state["epoch"] = epoch

    # Grow
    if random.random() < ADD_OBJECT_P: add_object(epoch)
    if random.random() < ADD_MORPHISM_P: add_morphism(epoch)

    # Emergence
    detect_triangle(epoch)
    before = len(state["two_cells"])
    try_create_2cell_from_conflict(epoch)
    after  = len(state["two_cells"])

    # Attractors
    det.record_endpoint_conflicts(epoch)
    if after > before: det.record_two_cell(epoch)
    det.check_and_flag(epoch)

    # Diamonds
    dia.scan(epoch)
```

```
    for hit in dia.check_remarkable(epoch):
        key = "diamond_loop" if hit["type"].endswith("loop") else
"diamond_nonloop"
        if not remarkable[key]:
            remarkable[key] = hit
            log_event(epoch, hit["type"], hit["detail"])


    # Compression signal
    comp = avg_distinct_paths_per_endpoint()
    if prev_comp is not None:
        rel = (comp - prev_comp) / (prev_comp + 1e-9)
        if (rel >= PHASE_JUMP_RELATIVE or comp >= PHASE_ABS_THRESHOLD) and
remarkable["phase_jump"] is None:
            remarkable["phase_jump"] = {"epoch": epoch,
"type":"remarkable:phase_jump",
                                        "detail": f"Compression jump: prev={prev_comp:.2f}
now={comp:.2f} (rel={rel:.2f})"}
            log_event(epoch, "remarkable:phase_jump",
remarkable["phase_jump"]["detail"])
    prev_comp = comp
    if epoch % PRINT_COMPRESSION_EVERY == 0:
        log_event(epoch, "signal:compression",
f"avg_distinct_paths_per_endpoint={comp:.2f}")


    # Broad stability
```

```
        stable_now = det.stable_endpoint_count_now(epoch)
        if stable_now >= REMARKABLE_BROAD_ATTRACTORS and
remarkable["broad_stability"] is None:
            remarkable["broad_stability"] = {"epoch": epoch,
"type":"remarkable:broad_stability",
                                             "detail": f"{stable_now} endpoint attractors stable within
window"}
            log_event(epoch, "remarkable:broad_stability",
remarkable["broad_stability"]["detail"])


        # Persist
        save_state(epoch)
        epoch += 1
        time.sleep(0.015)  # gentle pacing


    # Final
    remarkable["final"] = {"epochs": epoch,
                           "objects": len(state["objects"]),
                           "morphisms": len(state["morphisms"]),
                           "two_cells": len(state["two_cells"]),
                           "final_compression": comp,
                           "run_dir": os.path.abspath(RUN_DIR)}
    with open(SUMMARY, "w", encoding="utf-8") as f:
        json.dump(remarkable, f, indent=2)
```

```
    print("\n=== Long-run summary ===")
    for k,v in remarkable.items(): print(k, "=>", v)
    print("Logs in:", os.path.abspath(RUN_DIR))

if __name__ == "__main__":
    main()
```

Appendix 2. g_control.py

```
# g_control.py
# Control baseline: random directed multigraph growth with NO categorical coherence and NO 2-cells.
# Same logging/snapshot format as g_core.py so we can compare frequencies of motifs.

import json, os, random, time, datetime
from collections import defaultdict, deque

# ==== Config ====
RUN_SECS              = 60 * 60      # ~1 hour (match g_core)
RANDOM_SEED           = 424242
ADD_OBJECT_P          = 0.20
ADD_MORPHISM_P        = 0.55
BAN_SELF_MORPHISMS    = True
```

```
MAX_PATH_LEN           = 3
ATTR_WINDOW            = 20
ATTR_HITS_THRESH       = 6
ATTR_MIN_VARIANTS      = 2
PRINT_COMPRESSION_EVERY  = 60
PHASE_JUMP_RELATIVE     = 0.75        # +75% jump
PHASE_ABS_THRESHOLD     = 45.0
REMARKABLE_BROAD_ATTRACTORS = 20

# ==== Paths ====
RUN_TAG  = datetime.datetime.now().strftime("run-ctrl-%Y%m%d-%H%M%S")
BASE_DIR = "genesis_logs"
RUN_DIR  = os.path.join(BASE_DIR, RUN_TAG)
HIST_DIR = os.path.join(RUN_DIR, "history")
EVENTS   = os.path.join(RUN_DIR, "genesis_events.jsonl")
STATE_LATEST = os.path.join(RUN_DIR, "state_latest.json")
SUMMARY  = os.path.join(RUN_DIR, "remarkable_summary.json")
os.makedirs(HIST_DIR, exist_ok=True)

# ==== RNG ====
if RANDOM_SEED is not None:
   random.seed(RANDOM_SEED)
```

```
# ==== Logging ====
def log_event(epoch, kind, description, extra=None):
    rec = {"epoch": epoch, "type": kind.strip(), "description": description, "ts": time.time()}
    if extra: rec.update(extra)
    with open(EVENTS, "a", encoding="utf-8") as f:
        f.write(json.dumps(rec) + "\n")
    print(f"[Epoch {epoch}] {kind}: {description}")

# ==== Naming ====
def next_object_name(objs):
    n = len(objs) + 1
    name = ""
    while n > 0:
        n, rem = divmod(n - 1, 26)
        name = chr(65 + rem) + name
    return name

# ==== State (start size mirrors g_core) ====
state = {
    "epoch": 0,
    "objects": ["A","B","C"],
    # start with the same 3 arrows to keep initial density comparable
```

```
    "morphisms": [("A","B","f"), ("B","C","g"), ("A","C","h")],
    "two_cells": [],                # none in control
    "flags": {"first_triangle": False, "first_2cell": False}
}

# ==== Graph helpers ====
def adjacency():
    adj = defaultdict(list)
    for s,t,l in state["morphisms"]:
        adj[s].append((t,l))
    return adj

def paths(src, tgt, max_len=MAX_PATH_LEN):
    adj = adjacency()
    out, q = [], deque([(src, [], 0)])
    guard = 200000
    while q and guard > 0:
        guard -= 1
        node, path, d = q.popleft()
        if d >= max_len: continue
        for (nx, lbl) in adj[node]:
            step = (node, nx, lbl)
            newp = path + [step]
```

```
            if nx == tgt: out.append(newp)
            q.append((nx, newp, d+1))
    return out

def distinct_label_paths(plist):
    seen, uniq = set(), []
    for p in plist:
        sig = tuple(step[2] for step in p)
        if sig not in seen:
            seen.add(sig); uniq.append(p)
    return uniq

def avg_distinct_paths_per_endpoint():
    endpoints = set((s,t) for (s,t,_) in state["morphisms"] if s != t)
    counts = []
    for (A,C) in endpoints:
        uniq = distinct_label_paths(paths(A,C))
        if uniq: counts.append(len(uniq))
    return (sum(counts)/len(counts)) if counts else 0.0

# ==== Growth (pure random) ====
def add_object(epoch):
    name = next_object_name(state["objects"])
```

```
    state["objects"].append(name)
    log_event(epoch, "grow", f"New object {name}")

def add_morphism(epoch):
    if len(state["objects"]) < 2: return
    src, tgt = random.sample(state["objects"], 2)
    if BAN_SELF_MORPHISMS and src == tgt: return
    label = f"m{len(state['morphisms'])}"
    state["morphisms"].append((src,tgt,label))
    log_event(epoch, "compose", f"Morphism {src} → {tgt} ({label})")

# ==== “Emergence” detectors (no 2-cells in control) ====
def detect_triangle(epoch):
    if state["flags"]["first_triangle"]: return
    adj = adjacency()
    for A in state["objects"]:
        for (B, _) in adj[A]:
            for (C, _) in adjacency()[B]:
                if any(s==A and t==C for (s,t,_) in state["morphisms"]):
                    state["flags"]["first_triangle"] = True
                    log_event(epoch, "emergence:first_commutative_triangle",
f"{A}→{B}→{C} with direct {A}→{C}")
                    return
```

```
class AttractorDetector:
    def __init__(self, window=ATTR_WINDOW, hits_thresh=ATTR_HITS_THRESH):
        self.window = window; self.hits_thresh = hits_thresh
        self.endpoint_hits = defaultdict(list)

    def record_endpoint_conflicts(self, epoch):
        endpoints = set((s,t) for (s,t,_) in state["morphisms"] if s!=t)
        for (A,C) in endpoints:
            ps = distinct_label_paths(paths(A,C))
            if len(ps) >= ATTR_MIN_VARIANTS:
                self.endpoint_hits[(A,C)].append(epoch)

    def _stable(self, seq, now):
        return sum(1 for e in seq if e >= now - self.window)

    def check_and_flag(self, epoch):
        for k, epochs in list(self.endpoint_hits.items()):
            stab = self._stable(epochs, epoch)
            if stab >= self.hits_thresh:
                A,C = k
                log_event(epoch, "attractor:endpoint",
```

```
                    f"Stable multi-path endpoint {A}⇒{C} (hits last {self.window} =
{stab})")
            # don’t suppress repeats; we’ll summarize later

    def stable_endpoint_count_now(self, epoch):
        total = 0
        for _, hits in self.endpoint_hits.items():
            if self._stable(hits, epoch) >= self.hits_thresh:
                total += 1
        return total

class DiamondMonitor:
    """Detect loop and non-loop diamonds from raw edges (no lifts here)."""
    def __init__(self, window=ATTR_WINDOW, hits_thresh=ATTR_HITS_THRESH):
        self.window = window; self.hits_thresh = hits_thresh
        self.loop_hits = defaultdict(list)     # (A,B,C)->epochs
        self.nonloop_hits = defaultdict(list)  # (A,B,C,D)->epochs
        self.loop_first = None
        self.nonloop_first = None

    def scan(self, epoch):
        adj = adjacency()
        # loop: A->B->A and A->C->A
```

```
    for A in state["objects"]:
        for (B, _) in adj[A]:
            if any(t==A for (t,_) in adjacency()[B]):
                for (C, _) in adj[A]:
                    if C!=B and any(t==A for (t,_) in adjacency()[C]):
                        self.loop_hits[(A,B,C)].append(epoch)
    # non-loop: A->B->D and A->C->D with B!=C and D!=A
    for A in state["objects"]:
        for (B, _) in adj[A]:
            for (D, _) in adjacency()[B]:
                for (C, _) in adj[A]:
                    if C==B: continue
                    if any(t==D for (t,_) in adjacency()[C]) and D!=A:
                        self.nonloop_hits[(A,B,C,D)].append(epoch)


def check_remarkable(self, epoch):
    out = []
    cutoff = epoch - self.window
    if self.loop_first is None:
        for k, times in self.loop_hits.items():
            if sum(e>=cutoff for e in times) >= self.hits_thresh:
                self.loop_first = {"epoch": epoch, "type": "remarkable:diamond_loop",
```

```
                                "detail": f"Diamond stabilized: {k[0]}→{k[1]}→{k[0]} and
{k[0]}→{k[2]}→{k[0]}"}
                out.append(self.loop_first); break
        if self.nonloop_first is None:
            for k, times in self.nonloop_hits.items():
                if sum(e>=cutoff for e in times) >= self.hits_thresh:
                    A,B,C,D = k
                    self.nonloop_first = {"epoch": epoch, "type":
"remarkable:diamond_nonloop",
                                          "detail": f"Diamond stabilized: {A}→{B}→{D} and
{A}→{C}→{D}"}
                    out.append(self.nonloop_first); break
        return out


# ==== Persistence ====
def save_state(epoch):
    snap = {
        "epoch": epoch,
        "objects": state["objects"],
        "morphisms": state["morphisms"],
        "two_cells": state["two_cells"],   # stays empty
        "flags": state["flags"]
    }
    with open(STATE_LATEST, "w", encoding="utf-8") as f:
```

```
        json.dump(snap, f, indent=2)
    with open(os.path.join(HIST_DIR, f"state_{epoch:05d}.json"), "w",
encoding="utf-8") as f:
        json.dump(snap, f)


# ==== Main loop ====
def main():
    print(f"[Control] starting → {os.path.abspath(RUN_DIR)}")
    start = time.time()
    epoch = 0
    det   = AttractorDetector(window=ATTR_WINDOW,
hits_thresh=ATTR_HITS_THRESH)
    dia   = DiamondMonitor(window=ATTR_WINDOW,
hits_thresh=ATTR_HITS_THRESH)

    remarkable = {"diamond_loop": None, "diamond_nonloop": None,
"phase_jump": None,
                  "broad_stability": None, "final": {}}
    prev_comp = None

    while time.time() - start < RUN_SECS:
        state["epoch"] = epoch

        # Random growth only
```

```
    if random.random() < ADD_OBJECT_P: add_object(epoch)
    if random.random() < ADD_MORPHISM_P: add_morphism(epoch)

    # Triangle (for parity with g_core logging)
    detect_triangle(epoch)

    # Attractors from path diversity (no 2-cells in control)
    det.record_endpoint_conflicts(epoch)
    det.check_and_flag(epoch)

    # Diamonds (purely from edges)
    dia.scan(epoch)
    for hit in dia.check_remarkable(epoch):
        key = "diamond_loop" if hit["type"].endswith("loop") else
"diamond_nonloop"
        if not remarkable[key]:
            remarkable[key] = hit
            log_event(epoch, hit["type"], hit["detail"])

    # Compression signal
    comp = avg_distinct_paths_per_endpoint()
    if prev_comp is not None:
        rel = (comp - prev_comp) / (prev_comp + 1e-9)
```

```
        if (rel >= PHASE_JUMP_RELATIVE or comp >= PHASE_ABS_THRESHOLD) and
remarkable["phase_jump"] is None:

            remarkable["phase_jump"] = {"epoch": epoch,
"type":"remarkable:phase_jump",

                                        "detail": f"Compression jump: prev={prev_comp:.2f}
now={comp:.2f} (rel={rel:.2f})"}

            log_event(epoch, "remarkable:phase_jump",
remarkable["phase_jump"]["detail"])

    prev_comp = comp

    if epoch % PRINT_COMPRESSION_EVERY == 0:

        log_event(epoch, "signal:compression",
f"avg_distinct_paths_per_endpoint={comp:.2f}")


    # Broad stability

    stable_now = det.stable_endpoint_count_now(epoch)

    if stable_now >= REMARKABLE_BROAD_ATTRACTORS and
remarkable["broad_stability"] is None:

        remarkable["broad_stability"] = {"epoch": epoch,
"type":"remarkable:broad_stability",

                                         "detail": f"{stable_now} endpoint attractors stable within
window"}

        log_event(epoch, "remarkable:broad_stability",
remarkable["broad_stability"]["detail"])


    # Persist

    save_state(epoch)
```

```
        epoch += 1
        time.sleep(0.015)  # same pacing

    # Final
    remarkable["final"] = {"epochs": epoch,
                           "objects": len(state["objects"]),
                           "morphisms": len(state["morphisms"]),
                           "two_cells": len(state["two_cells"]),
                           "final_compression": comp,
                           "run_dir": os.path.abspath(RUN_DIR)}
    with open(SUMMARY, "w", encoding="utf-8") as f:
        json.dump(remarkable, f, indent=2)

    print("\n=== Control-run summary ===")
    for k,v in remarkable.items(): print(k, "=>", v)
    print("Logs in:", os.path.abspath(RUN_DIR))

if __name__ == "__main__":
    main()
```

Appendix 3. g_report.py

g_report.py

```
# Post-run summary/telemetry. No external deps.


import os, json, argparse, glob
from collections import Counter


def pick_latest_run(base="genesis_logs"):
    cand = sorted(glob.glob(os.path.join(base,"run-*")), key=os.path.getmtime,
reverse=True)
    return cand[0] if cand else None


if __name__ == "__main__":
    ap = argparse.ArgumentParser()
    ap.add_argument("--run-dir", default=None)
    args = ap.parse_args()


    run_dir = args.run_dir or pick_latest_run()
    if not run_dir or not os.path.exists(run_dir):
        raise SystemExit("Run directory not found.")
    events_path = os.path.join(run_dir, "genesis_events.jsonl")
    summary_path = os.path.join(run_dir, "remarkable_summary.json")
    hist = os.path.join(run_dir, "history")
```

```
    # Count events
    n=0; types=Counter(); first_triangle=None; first_2cell=None
    if os.path.exists(events_path):
        with open(events_path,"r",encoding="utf-8") as f:
            for line in f:
                n+=1
                try: rec=json.loads(line)
                except: continue
                types[rec.get("type","?")] += 1
                if rec.get("type","").startswith("emergence:first_commutative_triangle") and first_triangle is None:
                    first_triangle = rec.get("epoch")
                if rec.get("type","").startswith("emergence:first_2cell_created") and first_2cell is None:
                    first_2cell = rec.get("epoch")
    snaps = sorted(glob.glob(os.path.join(hist,"state_*.json")))
    last = {}
    if snaps:
        with open(snaps[-1],"r",encoding="utf-8") as f:
            last = json.load(f)

    print("\n=== GenesisBoard Run Report ===")
```

```
    print(f"Events: {n}  |  Epochs: {snaps[0][-10:-5]}..{snaps[-1][-10:-5] if snaps else
'?'}")
    print("Top event types:", dict(types.most_common(10)))
    print("First triangle epoch:", first_triangle)
    print("First 2-cell epoch:  ", first_2cell)
    if last:
        print(f"Objects now: {len(last.get('objects',[]))}  |  Morphisms now:
{len(last.get('morphisms',[]))}  |  2-cells recorded: {len(last.get('two_cells',[]))}")
    if os.path.exists(summary_path):
        with open(summary_path,"r",encoding="utf-8") as f:
            summ=json.load(f)
        # small pretty
        print("\nRemarkable summary:")
        for k in
["diamond_loop","diamond_nonloop","phase_jump","broad_stability"]:
            if summ.get(k): print(f"  {k}: {summ[k]}")
        if summ.get("final"): print(f"\nFinal: {summ['final']}")
    print(f"\nSaved in: {os.path.abspath(run_dir)}")
```